



The $(cc) - (\overline{c}\overline{c})$ (Diquark-Antidiquark) States in e^+e^- Annihilation

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ABSTRACT

The mass spectrum and decay modes of the $(cc) - (\overline{c}\overline{c})$ states are estimated in a quark-gluon model. We argue that the peculiar resonance-like structures of $R(e^+e^- \rightarrow \text{hadrons})$ for $\sqrt{s} = 6-7$ GeV may be due to production of the P-wave $(cc) - (\overline{c}\overline{c})$ states. They are predicted to lie in the range 6.4 - 6.8 GeV and mainly decay into charmed mesons.



Charmonium molecules or $c\bar{c}q\bar{q}$ states¹ have been proposed to interpret some of the structures in e^+e^- annihilation cross section above 4 GeV. In a recent paper² we have studied various $c\bar{c}q\bar{q}$ states in a quark-gluon model. A rough estimate of the spectrum of the P-wave $b\bar{b}q\bar{q}$ states³ has also been made in anticipation of the experimental possibilities opened up by the CESR machine. In this paper we shall focus our attention on the $cc\bar{c}\bar{c}$ states not only for the completeness of our four quark model, but also for providing a possible explanation to the intriguing analysis that there might be some new phenomena or new particle production in e^+e^- annihilation for $\sqrt{s} = 6-7$ GeV, which was presented by Barnett et al.⁴ very recently. They find that the data for $R(e^+e^- \rightarrow \text{hadrons})$ lie at least 10% above the QCD theory in this energy region (see Fig. 1). In fact, the data show some possible resonance-like structures. The threshold effects for new particles such as new quarks, Higgs bosons, heavy leptons have been considered to interpret the discrepancy between the data and the theory. But none of them seems to be particularly attractive.

Here we shall consider the possibility that these resonance-like structures are due to production of the P-wave double-charmed diquark antidiquark systems, i.e. $cc - \bar{c}\bar{c}$ states. Employing the same model as used for $c\bar{c}q\bar{q}$ states,² we estimate the spectrum for these states. We find that they lie in the energy range $\sqrt{s} = 6.4 - 6.8$ GeV. They mainly decay into charmed meson pair. Their widths could be of the order of a few tens of MeV. The branching ratio for decay into $\psi(\eta_c\eta_c)$ should be very small, say 10^{-3} . The leptonic decay widths are also estimated. As a result, while our interpretation for the peculiar resonance structures of R seems compatible with the limited data, the non-resonant enhancement of R still remains a question and further experimental tests are needed.

1. MODEL AND SPECTRUM

As in the case of light quark $q\bar{q}q\bar{q}$ system,⁵ a $cc\bar{c}\bar{c}$ system can be separated into two clusters by the angular momentum barrier. We call a system diquarkonium when the cluster is a diquark. There are two types of diquarkonium in terms of color configuration, i.e. $(cc)_{\underline{3}}^* - (\bar{c}\bar{c})_{\underline{3}}$ and $(cc)_{\underline{6}} - (\bar{c}\bar{c})_{\underline{6}}^*$. We only consider those diquarks in which two quarks are in a relative S-wave. Because two quarks must be antisymmetric under simultaneous exchange of color, spin, and flavor labels, $(cc)_{\underline{3}}^*$ diquark must have spin 1 whereas $(cc)_{\underline{6}}$ diquark must have spin 0.

We now look at an elongated MIT bag formed by a diquark (cc) and an antidiquark $(\bar{c}\bar{c})$ (see Fig. 2). In the adiabatic (Born-Oppenheimer) approximation,⁶ the (cc) and $(\bar{c}\bar{c})$ can be treated as two static sources with opposite color charges of the gluon fields. As in the case of charmonium, there will exist a linear confining potential between two clusters:

$$V(r) = \lambda r \quad (1)$$

$$\lambda \propto \sqrt{C} \quad (2)$$

where C is the color $SU(3)$ Casimir operator eigenvalue for the cluster. Solving the Schrödinger equation with this potential for a diquarkonium, we find the energy level E_L (L is the angular momentum) to be

$$E_L = E_0 + \left(\frac{\lambda^2}{M_d} \right)^{1/3} (\zeta_L - \zeta_0) \quad , \quad (3)$$

where M_d is the effective mass of the diquark, $E_0 \sim 2M_d$, $\zeta_0 = 2.338$, $\zeta_1 = 3.361, \dots$ For a $(cc)_{\underline{3}}^* - (\bar{c}\bar{c})_{\underline{3}}$ state, λ takes the same value as in the charmonium: $\lambda \sim 0.24 \text{ GeV}^2$, which is determined from the charmonium spectrum with the effective mass

of the charmed quark $\mu_c \sim 1.533 \text{ GeV}$ (see later). For a $(cc)_{\underline{6}}^*$ state, according to (2)

$$\lambda = 0.24 \times \sqrt{\frac{C(\underline{6})}{C(\underline{3})}} = 0.24 \times \sqrt{\frac{10/3}{4/3}} \sim 0.38 \text{ GeV}^2.$$

Next we estimate the diquark mass M_d . We consider the interaction between two quarks within a diquark. Since they are close together, one-gluon exchange contribution may be a good approximation according to asymptotic freedom. In this case the color electric interaction takes the form

$$\frac{\delta}{4} \sum_{a=1}^8 \lambda_i^a \lambda_j^a \quad (4)$$

while the color magnetic interaction is given by

$$-\beta \sum_{a=1}^8 \lambda_i^a \lambda_j^a \vec{\sigma}_i \cdot \vec{\sigma}_j \quad (5)$$

where λ_i^a ($\vec{\sigma}_i$) are the color (spin) matrices for the i -th quark, δ and β are proportional to the color fine structure constant α_s , also related to the quark wave function. Then, adding the color electric and magnetic interactions to the "free" quark masses μ_c^0 , for a spin 1 $(cc)_{\underline{3}}^*$ diquark we find

$$M_d = 2\mu_c + \frac{2}{3}\delta + \frac{8}{3}\beta, \quad (6)$$

while for a spin 0 $(cc)_{\underline{6}}$ diquark

$$M_d = 2\mu_c + \frac{5}{3}\delta + 4\beta, \quad (7)$$

where $\mu_c = \mu_c^0 - 2/3\delta$, being the effective mass of the charmed quark. On the other hand, using (4) and (5) for a $(c\bar{c})_1$ state, we get

$$m_\psi = 2\mu_c + \frac{16}{3}\beta \quad (8)$$

$$m_{\eta_c} = 2\mu_c - 16\beta. \quad (9)$$

Taking $m_\psi = 3.095$ GeV and $m_{\eta_c} = 2.98$ GeV, μ_c and β are then determined: $\mu_c \sim 1.533$ GeV, $\beta \sim 5.4$ MeV. Furthermore, δ may be determined by the mass differences between possible baryonium states² or by the mass differences between ordinary mesons and baryons in the framework of the quark-diquark model.⁷ We believe a reasonable value for δ should be in the range 0-0.15 GeV. Here we take $\delta = 0.084$ GeV, the same value as used in the $c\bar{c}q\bar{q}$ states,² as a tentative choice.

With above parameters λ , μ_c , δ , β and eq. (3), we are in a position to calculate the spectrum for these diquarkonium states. The results are shown in Table 1. In particular, for the P-wave we predict two $J^{PC} = 1^{--} (cc)_3^* - (\bar{c}\bar{c})_3$ states at 6.55 GeV (with total spin $s = 0, 2$, respectively). However, after taking account of the spin-dependent forces between two clusters, one of them ($s = 2$) will lower to 6.39 GeV. The spin-dependent forces are simply estimated by assuming² that the potential which generates the spin-orbit and tensor forces between $(cc)_3^*$ and $(\bar{c}\bar{c})_3$ is the same as that between c and \bar{c} in charmonium and using the empirical evaluation of the expectation values for these forces from the observed mass splittings of the P-wave charmonium states and omitting the mass dependence of these forces.⁸ Moreover, we also predict an $L = 1$, $J^{PC} = 1^{--} (cc)_6 - (\bar{c}\bar{c})_6^*$ state at 6.82 GeV (total spin $s = 0$).

Although there may be some uncertainties in our mass estimation which will be shortly discussed we believe our predicted masses make sense within the limits of 200 MeV.

2. THE DECAY

In general, $(cc)\underline{3}^* - (\overline{cc})\underline{3}$ states mainly decay into the double-charmed baryon (ccq) pair by creation of a light quark pair

$$(cc)\underline{3}^* - (\overline{cc})\underline{3} \rightarrow (ccq)\underline{1} + (\overline{ccq})\underline{1} \quad . \quad (10)$$

However, for our $L = 1$, $J^{PC} = 1^{--}$ diquarkonia the predicted masses are below the threshold of double-charmed baryon pair (≥ 7 GeV) and consequently they have to decay into the charmed meson pair via a charmed quark pair annihilation followed by a light quark pair creation (Fig. 3(a))

$$(cc)\underline{3}^* - (\overline{cc})\underline{3} \rightarrow (c\overline{q})\underline{1} + (\overline{c}q)\underline{1} \\ (D\overline{D}, D\overline{D}^*, D^*\overline{D}^*, F\overline{F}, F\overline{F}^*, F^*\overline{F}^*) \quad . \quad . \quad (11)$$

These decays involve a much weaker suppression factor than that in ψ decay, since they can proceed via one gluon contribution whereas ψ decay requires three gluons. In Ref. 2 we have considered the resonance at 4.16 GeV in e^+e^- annihilation as a possible $c\overline{c}q\overline{q}$ state whose decay proceeds via light quark pair annihilation and subsequent creation (Fig. 3(b)). Comparing Fig. 3(a) with Fig. 3(b) we see that there should be a suppression factor for $c\overline{c}c\overline{c} \rightarrow D\overline{D}(D\overline{D}^*, D^*\overline{D}^*)$ relative to $c\overline{c}q\overline{q} \rightarrow D\overline{D}(D\overline{D}^*, D^*\overline{D}^*)$, which means that to annihilate (or create) a heavy quark

pair is more difficult than that of a light quark pair. This factor can be understood in a QCD picture as α_s at large quark mass square is smaller than at small mass square. Phenomenologically this factor may be taken as $(\mu_u/\mu_c)^2 \sim 1/25$.^{9,2} But this suppression can be compensated by the phase space factor $(= (1 - 4m_D^2/s)^{3/2})$. We then have

$$\frac{\Gamma(cc\bar{c}\bar{c} \rightarrow D\bar{D})}{\Gamma(c\bar{c}q\bar{q} \rightarrow D\bar{D})} \sim \left(\frac{1 - \frac{4m_D^2}{s_{cc}}}{1 - \frac{4m_D^2}{s_{cq}}} \right)^{3/2} \left(\frac{\mu_u}{\mu_c} \right)^2 \sim 0.3 - 1, \quad (12)$$

where $s_{cc} = (6.6 \text{ GeV})^2$, $s_{cq} = (4.16 \text{ GeV})^2$, $m_D = 1.87 - 2 \text{ GeV}$, $\Gamma(4.16 \rightarrow D\bar{D}) \sim 60 \text{ MeV}$. So we may conclude that the decay width for $J^{PC} = 1^{--} cc\bar{c}\bar{c}$ states could be of the order of a few tens MeV.

These $cc\bar{c}\bar{c}$ states can also decay into a charmed baryon (cqq) pair, but these decays should be more suppressed since two light quark pair creation is needed.

The $J^{PC} = 1^{--} cc\bar{c}\bar{c}$ states cannot simply decay into $\psi\psi(\eta_c\eta_c)$ by dissociation since there is an angular momentum barrier between two clusters. These decays can only occur via charmed quark pair rearrangement or annihilation (Fig. 3(c)) and should be greatly suppressed because of the involvement of one more factor $(\mu_u/\mu_c)^2$ and the small phase space. As a result,

$$\text{B.R. } (c\bar{c}\bar{c}\bar{c} \rightarrow \psi\bar{\psi}) \approx \frac{\Gamma(c\bar{c}\bar{c}\bar{c} \rightarrow \psi\bar{\psi})}{\Gamma(c\bar{c}\bar{c}\bar{c} \rightarrow D\bar{D})}$$

$$\approx \left(\frac{1 - \frac{4m_\psi^2}{s_{cc}}}{1 - \frac{4m_D^2}{s_{cc}}} \right)^{3/2} \left(\frac{\mu_u}{\mu_c} \right)^2 \approx 3 \times 10^{-3} \quad (13)$$

The $J^{PC} = 1^{--}$ $c\bar{c}\bar{c}\bar{c}$ states may also decay by emitting isospin zero mesons (ω , for instance) into an S-wave $c\bar{c}\bar{c}\bar{c}$ state ($J^{PC} = 0^{++}, 1^{++}, 2^{++}$) which will in turn readily decay into $\psi\bar{\psi}$ ($\eta_c \bar{\eta}_c$) (Fig. 4). However, they are OZI-forbidden processes and their branching ratio should be very small.

3. THE COUPLING TO THE PHOTON

We have no reliable way to estimate the leptonic decay width Γ_e for these $J^{PC} = 1^{--}$ $c\bar{c}\bar{c}\bar{c}$ states. We know that in the $(cc) - (\bar{c}\bar{c})$ system the diquark looks more point-like at long distances but not point-like at small distances since the diquarks are essentially extended objects. Nevertheless, for simplicity we shall treat diquarks as point-like bosons to make a very rough estimate of Γ_e . As in Ref. 2 we get

$$\Gamma_e((cc) - (\bar{c}\bar{c})) = \frac{24\alpha^2}{m^4} e_{cc}^2 |R'_P(0)|^2 \quad , \quad (14)$$

where $R'_P(0)$ is the derivative of the P-wave radial wave function at origin for the diquarkonium. Assuming that the wave function here is the same type as that for $c\bar{c}$ system, then by scaling arguments,¹⁰ $|R'_P(0)|^2 \propto \mu^3$ (μ is the reduced mass of the constituent particles), we get

$$|R'_P(0)|^2 = \left(\frac{M_d}{u_c}\right)^3 |R'_P(0)|_\psi^2 \sim \left(\frac{3}{1.5}\right)^3 \times 0.067 = 0.54 \text{ GeV}^2, \quad (15)$$

where¹¹ $|R'_P(0)|_\psi^2 \sim 0.067 \text{ GeV}^2$. Setting the effective charge of the diquark $e_{cc} = 2e_c = 4/3$, $m = 6.6 \text{ GeV}$, from (14) we get $\Gamma_e((cc) - (\bar{c}\bar{c})) \sim 0.65 \text{ keV}$. This value is somewhat too large and will certainly be suppressed by the non-point-like effects of the diquarks. Hopefully, it may still be large enough to make the resonance observable.

4. DISCUSSIONS

The concept of diquark (or other clusters) in hadron structure is of great interest. Failure to observe certain baryon resonances might be a strong support for this picture.⁷ The MIT bag model has provided a theoretical description for the multiquark system. There have already been some experimental evidence⁵ for their existence. In e^+e^- physics, aside from the $J^{PC} = 1^{--} q\bar{q}q\bar{q}$,¹² $c\bar{c}q\bar{q}$ states, the $c\bar{c}c\bar{c}$ states are of particular interest, since they lie in the energy region far from any $q\bar{q}$ ($q = u, d, s, c, b$) states. Therefore the experimental evidences for the $c\bar{c}c\bar{c}$ states may establish the existence of multi-quark states in an unambiguous way.

We know that there are some uncertainties in our mass estimate such as the form of potential (linear, or logarithmic, or linear plus Coulombic), the involved parameters, the possible mixing between $(cc)_{\underline{3}}^* - (\bar{c}\bar{c})_{\underline{3}}$ and $(cc)_{\underline{6}} - (\bar{c}\bar{c})_{\underline{6}}^*$ states, etc. Especially, in our treatment we have ignored the extended structure of the diquark at small distances. Nevertheless, we believe that the $J^{PC} = 1^{--} c\bar{c}c\bar{c}$ states should lie in the range 6-7 GeV and that our analyses of their decays make sense. Thus these $c\bar{c}c\bar{c}$ states can be distinguished experimentally from other phenomena by their own properties. Since the production of $c\bar{c}c\bar{c}$ states in hadron-hadron collisions requires creation of two heavy quark pairs, it is almost hopeless to see

$c\bar{c}c\bar{c}$ states in these channels. In practice, the only opportunity of observing $c\bar{c}c\bar{c}$ states is still in e^+e^- annihilation. We hope more detailed data will be available in the near future. In addition, as a test of our model, a very careful measurement of R in the range $\sqrt{s} = 19-21$ GeV in search for the $J^{PC} = 1^{--} (b\bar{b}) - (\bar{b}b)$ states is also needed.

ACKNOWLEDGMENTS

I am grateful to Profs. N. Isgur, R.L. Jaffe, H. Lipkin, and C. Quigg for helpful discussions. In particular I would like to thank Prof. C. Quigg for reading the manuscript and offering valuable suggestions and for the warm hospitality extended to me during my visit at the Theory Division of Fermilab.

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- ⁸ For instance, the matrix elements for $\vec{L} \cdot \vec{S}$ force are proportional to $1/m^2 \langle 1/r \, dV/dr \rangle$. For the short-ranged forces (dominated by Coulomb potential) the mass dependence of $\langle 1/r \, dV/dr \rangle$ tends to compensate the factor $1/m^2$, so we may assume matrix elements are mass independent.
- ⁹ This suppression factor has been discussed in many literatures. See, for instance: A. Le Yaouanc, L. Oliver, O. Pene and J.C. Raynal, Phys. Lett. 72B (1977) 57.
- ¹⁰ For the power-law potential $V(r) = Ar^\nu$, $|d^\ell R_\ell(r)/dr^\ell|_{r=0}^2 \propto \mu^{(3+2\ell/2+\nu)}$. Roughly speaking, the level spacing data for ψ and T imply $\Delta E \propto \mu^0$, which leads to $\nu = 0$, $|R'_p(0)|^2 \propto \mu^{5/2}$; whereas the $\Gamma_{e^+e^-}$ data imply $|R_S(0)|^2 \propto \mu^2$, which leads to $\nu = -1/2$, $|R'_p(0)|^2 \propto \mu^{10/3}$. For simplicity we shall take the average: $|R'_p(0)|^2 \propto \mu^3$. See: C. Quigg and J.L. Rosner, Phys. Rep. 56C (1979) 169.
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Table 1(a), The quantum numbers and masses for the $(cc)_3^* - (\bar{c}\bar{c})_3$ states (without spin-dependent forces between two clusters)

L	S	J^{PC}	Mass (GeV)
1	0	1^{--}	6.55
	1	$0^{++}, 1^{+-}, 2^{+-}$	
	2	$1^{--}, 2^{--}, 3^{--}$	
2	0	2^{++}	6.78
	1	$1^{+-}, 2^{+-}, 3^{+-}$	
	2	$0^{++}, 1^{++}, 2^{++}, 3^{++}, 4^{++}$	
3	0	3^{--}	6.98
	1	$2^{+-}, 3^{+-}, 4^{+-}$	
	2	$1^{--}, 2^{--}, 3^{--}, 4^{--}, 5^{--}$	

Table 1(b). The quantum numbers and masses for the $(cc)_6 - (\bar{c}\bar{c})_6^*$ states (without spin-dependent forces between two clusters)

L	S	J^{PC}	Mass (GeV)
1	0	1^{--}	6.82
2	0	2^{++}	7.15
3	0	3^{--}	7.41

FIGURE CAPTIONS

- Fig. 1: Data for R from the SLAC-LBL collaboration. The curve is the QCD prediction for R (Ref. 4).
- Fig. 2: An elongated bag. The diquark and antidiquark are linked by color electric flux.
- Fig. 3: (a) The $(cc) - (\overline{c}\overline{c})$ states decay into charmed meson pairs.
 (b) The $(cq) - (\overline{c}\overline{q})$ states decay into charmed meson pairs.
 (c) The $(cc) - (\overline{c}\overline{c})$ states decay into $\psi\psi$ ($\eta_c\eta_c$).
- Fig. 4: The P-wave $(cc) - (\overline{c}\overline{c})$ states decay into S-wave $cc\overline{c}\overline{c}$ states by emitting light mesons.

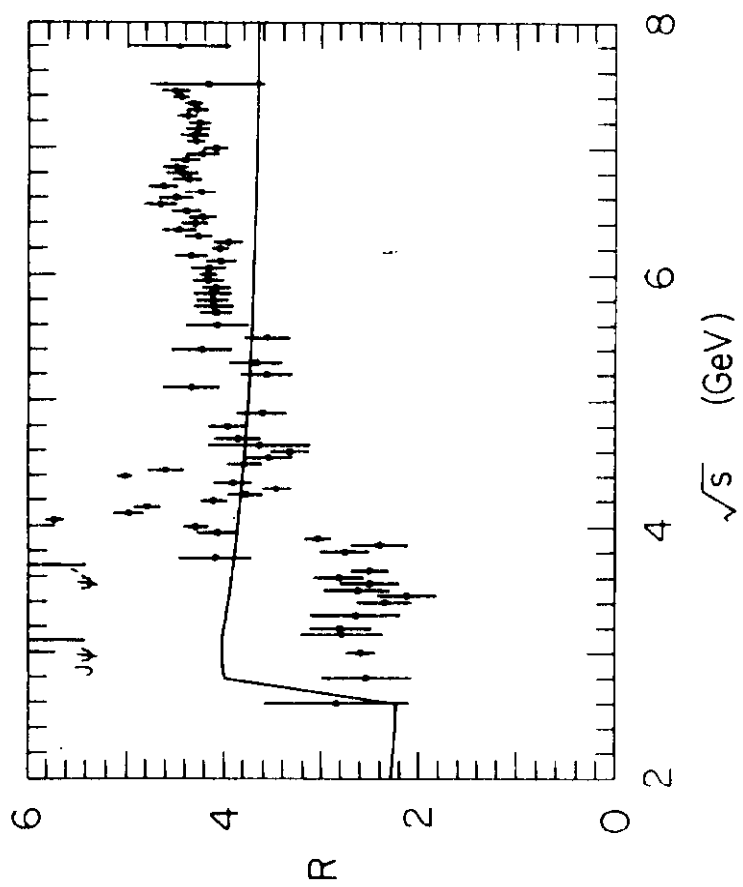


Fig. 1

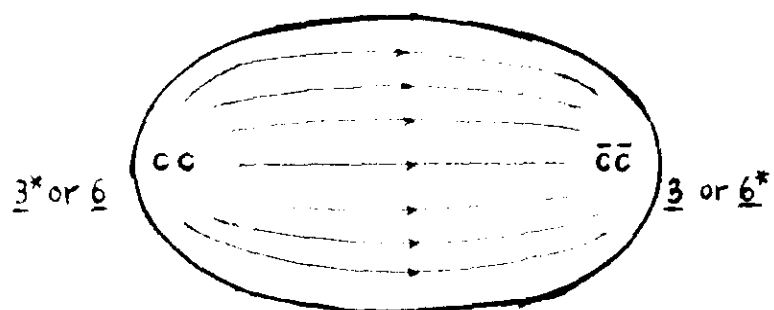


Fig. 2

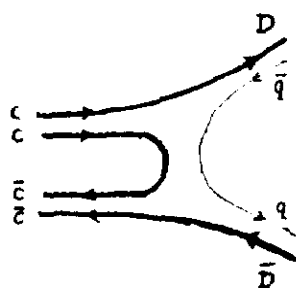


Fig. 3(a)

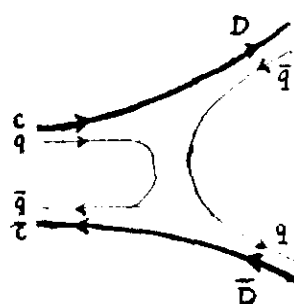


Fig. 3(b)

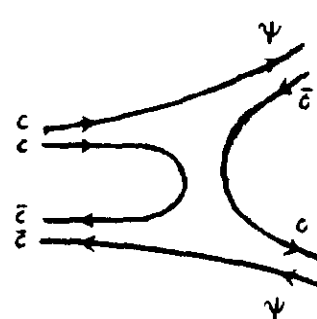


Fig. 3(c)

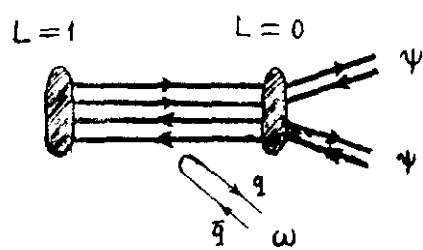


Fig. 4